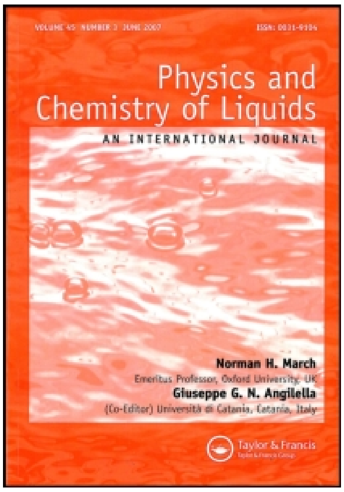

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Gordon R. Freeman[a]; Norman H. March[b]
[a] Department of Chemistry, University of Alberta, Edmonton, Alberta, Canada [b] Theoretical Chemistry Department, University of Oxford, Oxford, England

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LETTER

Steady-state Current Density Represented by a Velocity Vector Potential in an Inhomogeneous Electron Fluid in a Magnetic Field

GORDON R. FREEMAN

Department of Chemistry, University of Alberta, Edmonton, Alberta, Canada T6G 2G2

and

NORMAN H. MARCH

Theoretical Chemistry Department, University of Oxford, 5 South Parks Road, Oxford OX1 3UB, England

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For a steady-state current density **J**(**r**), the continuity equation reduces to div **J** = 0. In analogy with the description of a magnetic field **B** by a vector potential **A**, we propose to describe **J**(**r**) by a velocity vector potential **V**(**r**). Naturally, as with **A**, **V** is arbitrary to within the addition of the gradient of any scalar. The utility of **V**(**r**) is illustrated by constructing it for a model of independent harmonically confined electrons in a constant magnetic field. The **r** dependence of **V**(**r**) is shown in this model to be completely determined by an equilibrium property, namely the Slater sum $Z(\mathbf{r}, \beta)$. By explicit construction of the velocity vector potential for an inhomogeneous electron fluid in the regime of weak magnetic field plus semiclassical mechanics, it is demonstrated that a nonequilibrium property, namely current density, is linked directly with the equilibrium Slater sum for somewhat more general axially symmetric potentials than the purely harmonic form.

KEY WORDS: Velocity vector potential, current, magnetic field.

In a previous study[1], we have calculated the current density **J**(**r**) for independent harmonically confined electrons in a constant magnetic field. The conclusion from that work is that a steady current is induced by a magnetic field on the nonuniform charge distribution characterizing this model of an inhomogeneous electron fluid.

Motivated by this conclusion, we wish here to propose the use of a velocity vector potential **V**(**r**) to describe any steady-state current density **J**(**r**). For such a steady-state situation, the usual continuity equation relating div **J**(**r**) to the time–derivative of the density reduces to

$$\operatorname{div} \mathbf{J}(\mathbf{r}) = 0. \tag{1}$$

We then work in complete analogy with the introduction of a vector potential **A** characterizing a magnetic field **B**. This latter quantity satisfies the Maxwell equation

$$\operatorname{div} \mathbf{B} = 0, \tag{2}$$

allowing **B** to be written in the form

$$\mathbf{B} = \operatorname{curl} \mathbf{A}. \tag{3}$$

Thus, we shall define the velocity vector potential $\mathbf{V}(\mathbf{r})$ introduced above by

$$\mathbf{J}(\mathbf{r}) = \operatorname{curl} \mathbf{V}(\mathbf{r}) \tag{4}$$

which automatically satisfies Eq. (1). Just as with **A** introduced in Eq. (3), $\mathbf{V}(\mathbf{r})$ defined through Eq. (4) is arbitrary to within the addition of the gradient of any scalar.

To illustrate the usefulness of $\mathbf{V}(\mathbf{r})$, let us return to the model of Ref. [1]. There, the current density **J** has the form

$$\mathbf{J} = J_x\mathbf{i} + J_y\mathbf{j} \tag{5}$$

with **i** and **j** the usual Cartesian unit vectors in the x and y directions, respectively. One then finds

$$J_x = -y[\phi(\beta) - \phi(\beta, k = 0)]Z(\mathbf{r}, \beta), \tag{6}$$

with an analogous expression for J_y. In Eq. (6), $Z(\mathbf{r}, \beta)$ is the Slater sum, k is the force constant of the harmonic potential, while $\phi(\beta)$ is the phase factor entering the off-diagonal Slater sum, which is the so-called canonical density matrix.

It is now a fairly straight matter to construct a particular $\mathbf{V}(\mathbf{r})$ from Eq. (4) which will reproduce the current density in Eq. (6). We take

$$\mathbf{V} = (0, 0, V_z) \tag{7}$$

and then find

$$V_z(\mathbf{r}) = \frac{[\phi(\beta) - \phi(\beta, k = 0)]}{8h(\beta)} Z(\mathbf{r}, \beta), \tag{8}$$

where $h(\beta)$ is the "nonhomogeneity factor" appearing in the model Slater sum:

$$Z(\mathbf{r}, \beta) = f(\beta) \exp[-4(x^2 + y^2)h(\beta)]. \tag{9}$$

What seems to us remarkable is that, in this particular model, the entire spatial dependence of $V_z(\mathbf{r})$ is contained in the Slater sum $Z(\mathbf{r}, \beta)$. In turn, through Eq. (4), this "equilibrium" quantity determines the spatial shape of the current density $\mathbf{J}(\mathbf{r})$. In the above model, the Slater sum $Z(\mathbf{r}, \beta)$ is simply a Gaussian in the (x, y) plane.

Prompted by the above exactly soluble model, we next enquire whether the link exposed between a nonequilibrium property $\mathbf{J}(\mathbf{r}, \beta)$ and the Slater sum $Z(\mathbf{r}, \beta)$ is robust enough to survive a generalization of the harmonic oscillator axially symmetric potential $\frac{1}{2}k(x^2 + y^2)$ to $\mathscr{V}(x^2 + y^2)$. Below we denote by $\mathscr{V}'$ the first derivative of $\mathscr{V}$ with respect to its argument $(x^2 + y^2)$. Instead of now specifying the form of $\mathscr{V}$, we shall explicitly construct the velocity vector potential in the regime of weak magnetic fields plus semiclassical mechanics. The latter constraint imposes conditions on the smallness of higher derivatives than $\mathscr{V}'$ of the potential.

Our starting point[2,3] is the result for the current density $\mathbf{J}(\mathbf{r}, \beta)$ in this weak field semiclassical regime:

$$\mathbf{J}(\mathbf{r}, \beta) = -\frac{\beta^2}{6} C_0(\beta) \exp[-\beta\mathscr{V}(\mathbf{r})]\mathbf{B} \times \nabla\mathscr{V}, \tag{10}$$

where $C_0(\beta)$ is the partition function for free electrons.

Writing again $\mathbf{J} = \text{curl}\,\mathbf{V}$, it is straightforward to verify that

$$\mathbf{V} = (0, 0, V_z), \tag{11}$$

where

$$V_z = \frac{\beta B}{12} C_0(\beta) \exp(-\beta\mathscr{V}). \tag{12}$$

J_x and J_y then follow in terms of $y\mathscr{V}'$ and $x\mathscr{V}'$, respectively, as in Eq. (10). It will be noted that, as is the case in the exactly soluble model of axially symmetric harmonic confinement and arbitrary field strength B, the velocity potential $\mathbf{V}$ is determined completely in its spatial dependence by the Slater sum $Z(\mathbf{r}, \beta)$, which now has the semiclassical, field-independent form $C_0(\beta) \exp[-\beta\mathscr{V}(\mathbf{r})]$.

In summary, we introduce here the concept of a velocity vector potential $\mathbf{V}(\mathbf{r})$ to characterize a steady-state current density $\mathbf{J}(\mathbf{r})$. As with the usual vector potential $\mathbf{A}(\mathbf{r})$ describing a magnetic field $\mathbf{B}(\mathbf{r})$, $\mathbf{V}(\mathbf{r})$ is arbitrary to within the addition of the gradient of any scalar. For the model of independent, harmonically confined electrons in a constant magnetic field, a form of $\mathbf{V}(\mathbf{r})$ is derived. Remarkably, $\mathbf{V}(\mathbf{r})$ in its entire $\mathbf{r}$ dependence is then characterized by the Slater sum $Z(\mathbf{r}, \beta)$ in this model. Thus the spatial shape of the current density $\mathbf{J}(\mathbf{r})$ is completely specified by the "equilibrium" Slater sum in this case of an inhomogeneous electron fluid in a magnetic field.

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